

# A Theory of Actual Causation

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- Consider any scenario with two actually occurring events.

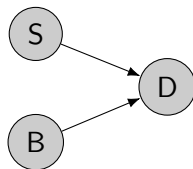
## Scenario

Suzy and Billy throw a rock at an empty bottle. The rocks hit the bottle at the same time and it breaks.

- A theory of actual causation is a simple algorithm that tells you whether one event caused another.
- The challenge: find a theory that aligns with the causal intuitions of professional philosophers, computer scientists and other academics.

## Scenario

Suzy (S) and Billy (B) throw a rock at an empty bottle. The rocks hit the bottle at the same time and it is destroyed (D). Either rock is sufficient for the rock breaking.



Equations:

$$D \doteq S \vee B$$

Valuation:

$$S \doteq 1$$

$$B \doteq 1$$

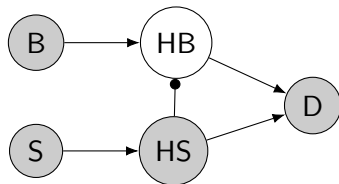
## Definition (Intervention)

An *intervention* on  $X \doteq x$  sets  $X$  to  $x$  and removes the equation for  $X$ .

- Almost all existing theories violate some intuitions, including Halpern (2015), Hall (2007), Beckers (2021), Halpern and Hitchcock (2015), and Gallow (2021).
- Andreas and Günther (forthcoming) seems to satisfy all intuitions.
- My theory makes the same judgments as Andreas and Günther (forthcoming), but it is simple, intuitive and counterfactual.

## Scenario

Billy (B) and Suzy (S) throw rocks at a bottle. Suzy's rock hits the bottle first (HS), shattering it (D). If Suzy's rock had not hit the bottle, then Billy's rock would have hit it (HB) and shattered it.



Equations:

$$HB \doteq B \wedge \neg HS$$

$$HS \doteq S$$

$$D \doteq HB \vee HS$$

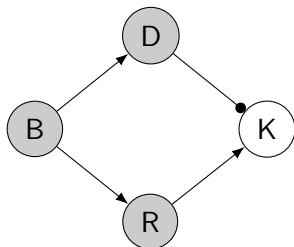
Valuation:

$$B \doteq 1$$

$$S \doteq 1$$

## Scenario

A boulder is dislodged (B) and rolls towards a hiker (R). The hiker sees the boulder dislodgement and ducks (D). As a result, the hiker survives. If the hiker had not ducked, he would have died (K).



Equations:

$$D \doteq B$$

$$R \doteq B$$

$$K \doteq R \wedge \neg D$$

Valuation:

$$B \doteq 1$$

## Definition (Model-Relative Causation)

$A \doteq a$  causes  $B \doteq b$  relative to a model  $M$  if and only if

1.  $A \doteq a$  and  $B \doteq b$  occur in  $M$ .
2. There exists an intervention set  $I_1$  which
  - intervenes on  $A \doteq a$ , and
  - may intervene on non-actual values of exogenous variables, and
  - may intervene on actual values of endogenous variables that are not a descendant of  $A$ .

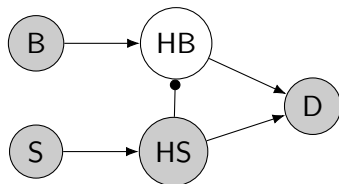
and an intervention set  $I_2$  which

- intervenes on  $A \doteq \neg a$ , and
- intervenes on the new value of every variable changed as a result of the intervention on  $I_1$ ,

such that

- (a)  $B \doteq b$  after intervening on  $I_1$ ,
- (b)  $B \doteq \neg b$  after intervening on both  $I_1$  and  $I_2$ .

# Preemption: does $B$ cause $D$ ? I



Equations:

$$HB \doteq B \wedge \neg HS$$

$$HS \doteq S$$

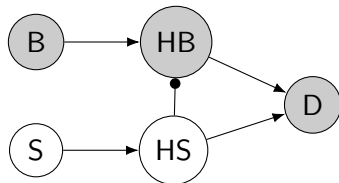
$$D \doteq HB \vee HS$$

Valuation:

$$B \doteq 1$$

$$S \doteq 1$$

- Let  $I_1 = \{B \doteq 1, S \doteq 0\}$ . This yields:



Equations:

$$HB \doteq B \wedge \neg HS$$

$$HS \doteq S$$

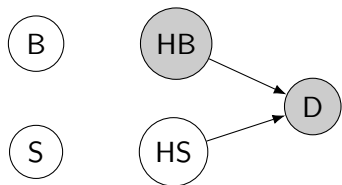
$$D \doteq HB \vee HS$$

Valuation:

$$B \doteq 1$$

$$S \doteq 0$$

- HB and HS have changed in value, so we have  $I_2 = \{B \doteq 0, HB \doteq 1, HS \doteq 0\}$ .
- Intervening on  $I_2 = \{B \doteq 0, HB \doteq 1, HS \doteq 0\}$  yields:



Equations:

$$HB \doteq B \wedge \neg HS$$

$$HS \doteq S$$

$$D \doteq HB \vee HS$$

Valuation:

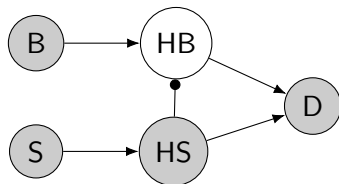
$$B \doteq 0$$

$$S \doteq 0$$

$$HB \doteq 1$$

$$HS \doteq 0$$

- In this model  $D$  is true, so  $B$  does not cause  $D$ .



Equations:

$$HB \doteq B \wedge \neg HS$$

$$HS \doteq S$$

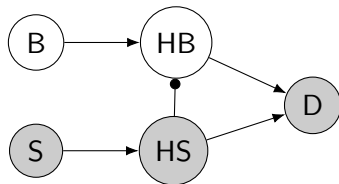
$$D \doteq HB \vee HS$$

Valuation:

$$B \doteq 1$$

$$S \doteq 1$$

- Let  $I_1 = \{S \doteq 1, B \doteq 0\}$ . Intervening yields:



Equations:

$$HB \doteq B \wedge \neg HS$$

$$HS \doteq S$$

$$D \doteq HB \vee HS$$

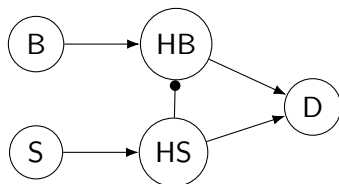
Valuation:

$$B \doteq 0$$

$$S \doteq 1$$

- Nothing has changed in value, so we have  $I_2 = \{S \doteq 0\}$ .

- Intervening on  $I_2 = \{S \doteq 0\}$  yields:



Equations:

$$HB \doteq B \wedge \neg HS$$

$$HS \doteq S$$

$$D \doteq HB \vee HS$$

Valuation:

$$B \doteq 0$$

$$S \doteq 0$$

- In this model  $D$  is false, so  $S$  causes  $D$ .

- A model  $M$  is appropriate for some scenario only if:
  - **Exogenous deviancy.** For every exogenous variable  $X$ , its actual value  $X \doteq x$  is more deviant than  $X \doteq \neg x$ .
  - **Exogenous saturation.** One cannot add a new exogenous variable  $X$  to  $M$  such that  $M + X$  is appropriate (in other senses).
  - **Endogenous saturation 1.** One cannot add a new endogenous variable  $X$  to  $M$ , such that  $X$  has at least two parents or two children and  $M + X$  is appropriate (in other senses).
  - **Endogenous saturation 2.** When a variable  $A$  in  $M$  has more than one parent or child and  $C$ , with no or two or more children, is a descendant of  $A$ , then  $M$  must have a variable  $B$  on the path between  $A$  and  $C$  with exactly one parent and child.
  - (Apart from the above, appropriateness requires accuracy and a number of other conditions.)

## Definition (Actual Causation 1)

An event **a** causes an event **b** in some scenario **S** if and only if **there exists** an appropriate model  $M$  such that  $A \doteq a$  causes  $B \doteq b$  with respect to  $M$ .

## Definition (Actual Causation 2)

An event **a** causes an event **b** in some scenario **S** if and only if **for all** appropriate models  $M$ ,  $A \doteq a$  causes  $B \doteq b$  with respect to  $M$ .

## Conjecture (Model-Invariance)

Let  $\mathbf{S}$  be a scenario and let  $M_1, M_2$  be appropriate models for  $\mathbf{S}$ . Then  $\mathbf{a}$  causes  $\mathbf{b}$  with respect to  $M_1$  if and only if  $\mathbf{a}$  causes  $\mathbf{b}$  with respect to  $M_2$ .

## Theorem (Local Model-Invariance)

*Let  $\mathbf{S}$  be a scenario and let  $M$  an appropriate model for  $\mathbf{S}$ . Let  $M + X$  be an appropriate model for  $\mathbf{S}$ , where the interpretation for the variables in  $M$  remains the same. Then  $\mathbf{a}$  causes  $\mathbf{b}$  in  $M$  if and only if  $\mathbf{a}$  causes  $\mathbf{b}$  in  $M + X$ .*

- My theory gives the intuitively correct judgments on the following types of scenario (and more):
  - Simple overdetermination
  - Late preemption
  - Early preemption
  - Threat and saviour (short circuit)
  - Double prevention
  - Preempted prevention
  - Switches
  - Bogus prevention
- My theory is simpler and more intuitive than similarly successful theories.

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